

A Coupled Simulated Annealing-Hill Approach for Mechanism Synthesis with Twenty Accuracy Points

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ABSTRACT

This paper proposes a novel approach for the Optimization of mechanism synthesis for function generation problem with twenty accuracy points using a hybrid search algorithm. A hybrid optimization method is based on the fusion of the Simulated annealing (SA) and Rosenbrock Search (RS), derivative-free type method of optimization, called SA/ RS, in which the SA is embedded the RS to enhance its search capability. The algorithm combines the advantages of the global optimization technique and a classical non linear programming technique. An overview of SA/ RS is presented and applied to dimensional synthesis of a planer four bar mechanism. The optimization is carried out to minimize the objective function formulated from the structural error at the accuracy points. A novel SA/ RS is employed to determine the optimal values for the design variables that best satisfy the desired objectives of the problem. Simulation results demonstrate the remarkable advantages of our approach in achieving the diverse optimal solutions and improved converge speed. The experimental results manifest that the proposed hybrid approaches are effective and efficient in finding near optimal solutions.

Keywords: Function generation, Hybrid algorithm, Rosenbrock Search, Simulated annealing.

ARTICLE INFO

Article History

Received : 29th February 2016

Received in revised form :

1st March 2016

Accepted : 4th March 2016

Published online :

6th March 2016

I. INTRODUCTION

In the past, a number of different techniques have been employed for the synthesis of mechanisms [1-4]. In the traditional approaches, the solution method is based on the graphical, and /or analytical design methods. Later with the proliferation of high speed computers and their integration into mechanism analysis and synthesis, a wide variety of numerical optimization methods have been developed for the synthesis of mechanisms. The graphical methods can provide a quick and easy method of design. But this approach has accuracy limitations. The analytical methods in practice today are mostly based on algebraic methods [5], displacement matrix method [6] or complex number methods [7]. In these approaches, the mechanism synthesis problem (MSP) is solved and carried out to satisfy accuracy

points exactly. A major drawback in using the analytical methods is that there could be significant error in the overall output between the precision points, branching and incorrect sequence of accuracy points and impractical for design and optimization of complex mechanism. Mechanism involving a finite number of links poses an inherent error and it is the task of the designer to reduce this error to a sufficiently low value. Many numerical optimizations [8-10] are available at present for design optimization of engineering problems to find optimum design. Solving MSP can be complex and a time consuming process when there are large numbers of design variables and constraints. Hence there is a need for efficient and reliable algorithms that solves such problems. It is known that no algorithm can surely find the absolute

minimum in a polynomial time with number of variables; some very successful heuristic algorithms have been developed. Amongst these, the SA, method of Kirkpatrick has proven to very successful in a broad class of situations.

SA is a stochastic heuristic algorithm in which the solutions are searched in hill climbing process constantly commenced by random moves. Because of its ease use, SA is an extremely popular method for solving large-sized and practical problems. However, for various reasons, like many other search algorithms, SA may become trapped by any local minima, which doesn't allow moving up or down, or take a long time to find a reasonable solution, which sometimes makes the method unpreferable. For this reason, many SA implementations have been done as a part of a hybrid method [11-22].

In this paper, SA has been developed and later on hybridizing externally with RS, the local search, provides the fastest solutions at the end of each generation. The Four bar function generator was optimally optimized using SA and also using SA/RS. On getting satisfactory results from SA, RS is activated and carried out local search to gain accuracy i.e. when SA procedure is in progress; a list of twenty best points is maintained and constantly updated, iff a new point is randomly created. At the termination of SA algorithm, these points are fed to RS algorithm as the starting points for the local search and new list of best points is created.

II. SIMULATED ANNEALING

A. Fundamental Concept:

SA [8-9] is a generic probabilistic Meta-algorithm for the global optimization problem, namely locating a good approximation to the global optimum of a given function in a large search space. SA is based on an analogy to the cooling of heated metals. In any heated metal sample the probability of some cluster of atoms at a position, (ri), exhibiting a specific energy, E (ri), at some temperature T, is defined by the Boltzmann Probability factor:

$$P(E(ri)) = e^{-[E(ri)/kBT]} \quad (1)$$

Where kB is Boltzmann's constant. As a metal is slowly cooled, atoms will fluctuate between relatively higher and lower energy levels and allowed to equilibrate at each temperature T. The material will approach a ground state, a highly ordered form in which there is a very little probability for the existence of a high energy state throughout the material. If the energy function of this physical system is replaced by an objective function, f(X), that is dependent on a vector of design variables, X, then the slow progression towards an ordered ground state is representative of a progression to a global optimum. To achieve this, a control temperature T, analogues to a temperature, and a constant C, analogues to kB, must be specified for the optimization problem. In standard iterative improvement methods, a series of trial points is generated until an improvement in the objective function is noted in which case the trial point is accepted. However, this process only allows for downhill movements to be made over the domain. In order to generate the annealing behavior, a secondary criterion is added to the process. If a trial point generates a large value of objective function then the probability of accepting this trial point is determined using the Boltzmann Probability distribution:

$$P[\text{accept } X_t] = e^{-[f(x_t) - f(x_0) / CT]} \quad (2)$$

Where x_0 is the initial starting point. This probability is compared against a randomly generated number over the range [0 1]. If $P[\text{accept } X_t] \geq \text{random}[0....1]$ then the trial point is accepted. This dependence on random numbers makes a SA a stochastic method.

B. SA Algorithm

The algorithm proceeds as follows:

Step1: Starting from the initial point X_0 , the algorithm generates successively improved points X_1, X_2, \dots moving towards the global minimum solutions. The initial value of a control parameter (T) is suitably high and a methodology for decrementing (T) is applied.

Step2: A sequence of design vector is then generated until equilibrium is reached.

Step3: During this phase the step vector (S) is adjusted periodically to better follow the function behavior. The best point is recorded as X_{opt} .

Step4: Once thermal equilibrium is reached, the temperature (T) is reduced and a new sequence of moves is made starting from X_{opt} . Until thermal equilibrium is reached again.

Step5: This process is continued until a sufficiently low temperature is reduced, at which stage no more improvement in the objective function value can be expected.

III. ROSENBROCK SEARCH

A. Fundamental Concept:

RS [8-10] is based on the "Automatic" method proposed by H.H.Rosenbrock. This method is a sequential search technique and solves the problem.

$$\text{Optimize } F(x_0(k), x_1(k), \dots, x_n(k)) \quad (3)$$

$$\text{Subject to } GK < XK < HK \quad (4)$$

The implicit variables $x_{N+1} \dots x_M$ are dependent functions of the explicit independent variables $x_0(k), x_1(k), \dots, x_n(k)$. The upper and lower constraints GK and HK are either constants or functions of the independent variables. The goal of RS is to search for the minimum of a nonlinear object function. It is an iterative procedure that bears some correspondence to the exploratory search of Hooke and Jeeve's in that small steps are taken during the search in orthogonal coordinates. However, instead of continually searching the coordinates corresponding to the directions of the independent variables, an improvement can be made after one cycle of coordinate search by lining the search direction up into orthogonal systems, with the overall step on the previous stage as the first building block for the new search coordinates. In this method, the coordinate system is rotated in each stage of minimization in such manner that the first axis is oriented towards the locally estimated direction of valley and all other directions are made mutually orthogonal and normal to the first one. Each step is tested for success i.e. from $x_1(k)$ if we take the step length $\lambda_1(k)$ to search $x_2(k)$ and if $f(x_2(k)) < f(x_1(k))$, then the step is treated as success. Otherwise it is a failure.

B. RS Algorithm:

Let the function to be minimized be a function of n variables. Selection of a set of initial step length $\lambda_1(k), \dots, \lambda_n(k)$ to be taken along the search directions $S_1(k), S_2(k), \dots$

$S_n(k)$ respectively forms the beginning of process. The procedure for k th stage is given below.

Step1: The set of $S_1(k), S_2(k), \dots, S_n(k)$ and the base points $x_0(k)$ are known at the beginning of k th stage. A step length $\lambda_1(k)$ in the direction $S_1(k)$ from the known base point is considered. If step is successful, the step width is increased, $\lambda_1(k)$ is multiplied by a factor(α), the new point is retained and a success is recorded. If step is a failure, the step width is decreased, $\lambda_1(k)$ is multiplied by a factor(β) and a failure is recorded. The values of α and β recommended by Rosenbrock are $\alpha=3$ and $\beta=-0.5$.

Step2: The search is continued along the direction $S_1(k), S_2(k), \dots, S_n(k)$ until at least one step has been successful and one step has failed in each of n directions.

Step 3: The new set of directions for use in Gram-Schmidt orthogonalisation process,

$S_1(k+1), \dots, S_n(k+1)$ are computed as under:

Compute the $A_{n \times n}$ matrix according to the relation:

$$A_{n \times n} = [A_1(k), A_2(k), \dots, A_n(k)] \quad (5)$$

$$A_1(k) = \lambda_1(k) S_1(k) + \lambda_2(k) S_2(k) + \dots + \lambda_n(k) S_n(k)$$

$$A_2(k) = \lambda_1(k) S_2(k) + \dots + \lambda_n(k) S_n(k) \quad (6)$$

$$A_n(k) = \lambda_1(k) S_n(k)$$

Where $A_1(k)$ is the vector from $x_0(k)$ to $x_0(k+1)$, $A_2(k)$ is the vector from $x_1(k)$ to $x_0(k+1)$, and so on. $A_1(k)$ represents the overall move from stage k to stage $(k+1)$, $A_2(k)$ represents the overall move less the progress made during the search in direction $s_1(k)$, etc. and $\lambda_i(k)$ be the algebraic sum of all the successful steps (the net distance moved) in the direction $S_i(k)$ during the k th stage.

$$(b) \text{Set } B_1(k) = A_1(k) \text{ and } s_1(k+1) = A_1(k) / \|A_1(k)\| \quad (7)$$

$$(c) \text{Find } B_n(k) = A_n(k) - \sum [(A_n(k) \cdot S_i(k+1)) S_i(k+1)] S_i(k+1) \\ \text{With } S_i(k+1) = B_n(k) / \|B_n(k)\| \quad (8)$$

Where $\|A_i(k)\|$ is the normal of $A_i(k)$

Step4: Take the best obtained in the present stage as the best point for the next stage. Set the new iteration number as $(k+1)$ and repeat the procedure.

Step5: Convergence is assumed after completing specified number of stages or after satisfying the condition $|\lambda_k(k)| \leq \epsilon$ for all n , where ϵ is a specified error limit.

IV. HYBRID OPTIMIZATION ALGORITHM

Any one algorithm may not be sufficient to provide a satisfactory solution for the given problem at hand. In recent years there has been a great deal of interest in the development of optimization algorithms that deal with the problem of finding a global minimum of a given continuous function. These algorithms were innovated to confront the rapid growth of many optimization problems in engineering. Meta-heuristics methods are considered to be acceptably good solvers of this problem. The power of meta-heuristic methods come from the fact they are robust and can deal successfully with a wide range of problem areas.

However, these methods, especially when they are applied to complex problems, suffer from the high computational cost due to their slow convergence. The main reason for this slow convergence is that these methods explore the global search space by creating random movements without using much local information about promising search direction. In

contrast, local search methods have faster convergence due to their using local information to determine the most promising search direction by creating logical movements. The local search methods can easily be entrapped in local minima. One approach that recently has drawn much attention is to combine meta-heuristic methods with local search methods to design more efficient methods with relatively faster convergence than the pure meta-heuristic methods. Moreover, these hybrid methods are not easily entrapped in local minima because they still maintain the merits of the meta-heuristic methods.

The reason for studying and developing a hybrid algorithm can be summarized as follows:

1. To improve the performance of the Non-traditional search computation (NTSC).

(Example: fast convergence speed and robustness) i.e. SA method usually suffers from slow convergence due its random nature of movements. Moreover, it also suffers from the difficulty in obtaining some required accuracy although it may quickly approach the neighborhood of the global minimum. Hence we have focused on the importance of creating the hybrid approach and importance of accelerating the final stage of SA by using the faster convergent method.

2. To improve the quality of the solutions obtained by the NTSC.

3. It achieves the searching not only globally but also locally.

4. To prevent the premature convergence.

In order to prevent the premature convergence, the coupling of SA and local search methods, to form hybrid SA/RS can be advantageous.

5. It searches larger regions of the solution space effectively using both SA-based global search capability with traditional search-based local search feature.

6. The goal of developing a Hybrid simulated Annealing approach is to use this algorithm to help in solving the problem of optimal synthesis of a mechanism for the function generation problem.

Achieving this goal will help in providing the best set of design variables, reducing the structural error and satisfying number of design constraints.

7. Modern meta-heuristics manage to combine exploration and exploitation search.

The exploration search seeks for new regions, and once it finds a good region, the exploitation search kicks in. However, since the two strategies are usually inter-wound, the search may be conducted to other regions before it reaches the local optima. As a result, many researchers suggest employing a hybrid strategy, which embeds a local optimizer in between the iterations of the meta-heuristics.

The diagram of scheme SA/RS is shown in Fig.1

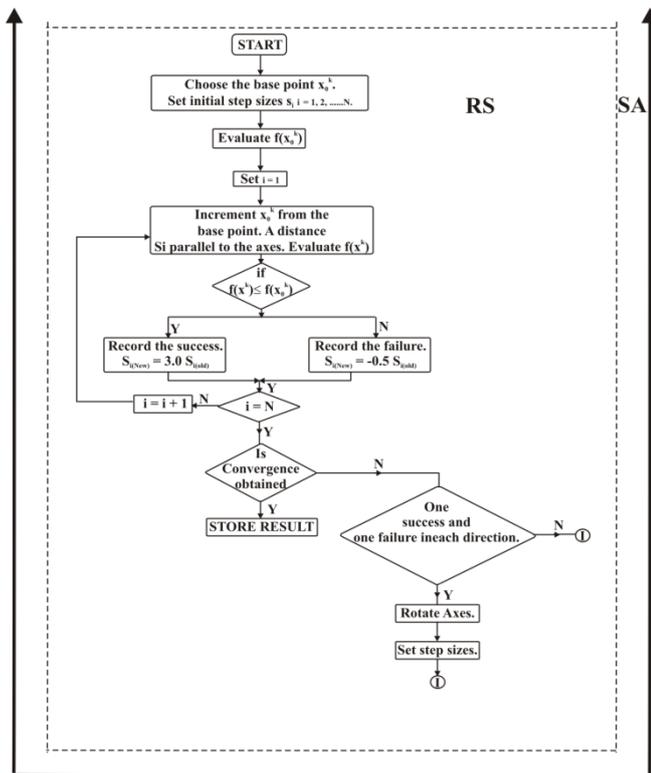


Fig. 5.2 Flow chart of Hybrid SADS

In this section, we developed a hybrid optimization algorithm based on the principles of both the aforementioned SA and RS. The SA method occasionally chooses those ‘uphill points’ from the current place. That is, not only the improved solutions but also the relatively weak ones are accepted with a specified probability according to different temperatures.

Thus the SA method has certain advantages, e.g., robustness and flexibility, over other local search methods, and is suitable for handling nonlinear problems. Unfortunately, it always takes a considerably long time to acquire the global optimum, because the temperature indeed needs to be decreased slowly enough during the iterations. A main advantage of creating a hybrid [18-22] of global optimization with traditional methods is to improve the global search and convergence speeds.

V. PROBLEM FORMULATION

This section address the problem of minimizing the error of function generating linkages under inequality constrains. Formulation of an optimal design problem involves identification of the design variables, X, objective function, f(X), and design constraints. Optimal synthesis procedure commonly minimizes the “structural error”, the error at a point in domain and is defined as the difference between the actual output displacement and the required output displacement. The design of mechanism can be formulated as a problem in non linear programming (NLP).

A. Objective Function:

Usually, in function generating mechanism design problem, a relation exists between the rotation angle of the input link, the expected angle of output link (Φ_{EXP}) and the generated angle of the output link ((Φ_{GEN}). The mean root value of error between (Φ_{EXP}) and (Φ_{GEN}) is generally used as f(X) to be minimize. The objective function, f(X), is taken as the sum of the squares of structural errors (in

radians) at different precision points. The implicit and the explicit constraints are incorporated as the penalty functions in the SA and RS Algorithm. The f(X) of the problem for minimisation can be expressed mathematically as:

$$f(X) = \sum_{\Theta=180^\circ/N}^{180^\circ} [(\Phi_{EXP} - (\Phi_{GEN}))]^2 \quad (9)$$

The output angle generated by the mechanism is considered as a function of input angle (Θ) in the following form:

$$(\Phi - \Phi_s)k_2 = f [(\Theta - \Theta_s) k_1] \quad (10)$$

$$\text{Where } f(\Theta) = A_1 \Theta^3 + A_2 \Theta^2 + A_3 \Theta + A_4 \quad (11)$$

Thus the expected value of output angle is calculated from the extension of above equation as

$$\Phi_{EXP} = [f [(\Theta - \Theta_s) k_1] / K_2 + \Phi_s] \quad (12)$$

Table I shows values of A₁, A₂, A₃ and A₄ used in above equations.

Table I
Values of function coefficients

Function Type	A ₁	A ₂	A ₃	A ₄
FunType1	3 x 10 ⁻⁶	5 x 10 ⁻⁴	0.385	65

Generated value of the output angle is a function of the link ratios X₁, X₂ and X₃ and input angle(Θ). The objective function for minimization is taken in (rad²) and in the form as given below :-

$$f(X) = \sum_{\Theta=180^\circ/N}^{180^\circ} [f(\Theta - \Theta_s) k_1 - (\Phi_{GEN} - \Phi_s) K_2]^2 \quad (13)$$

The values of input angle (Θ) are taken in the range of 180° / N to 180° in steps of 180° / N to produce N precision positions.

B. Design variables:

In this optimization problem, design variable vector X = [X₁, X₂, X₇], represents a solution that minimize the objective function. Parameters used in the formulation of objective function are as follows:

1. Ratio of Coupler/ Crank length = X₁
2. Ratio of Rocker / Crank length = X₂
3. Ratio of Fixed link/ Crank length = X₃
4. Initial starting angle of Crank link = X₄
5. Initial starting angle of Rocker = X₅
6. Scale for (Θ) = X₆
7. Scale for (Φ) = X₇

C. Constraints: The constraints considered for the optimum MSP are as follows:

1. 0.0 < X₁, X₂, X₃ ≤ 4.0
2. 0° ≤ X₄, X₅ ≤ 15°
3. 0.0 ≤ X₆, X₇ ≤ 2.0

VI. RESULTS AND DISCUSSION

A newly developed SA/RS has been tried on standard test problems for testing their effectiveness both on global optimization procedure conversely for faster and closer convergence. The results of this simulation are obtained by varying the seed values, number of iteration, and number of precision points. Many solutions were obtained globally and locally. The number of best solutions, number of iteration,

and number of precision points selected for the presentation are as follows:

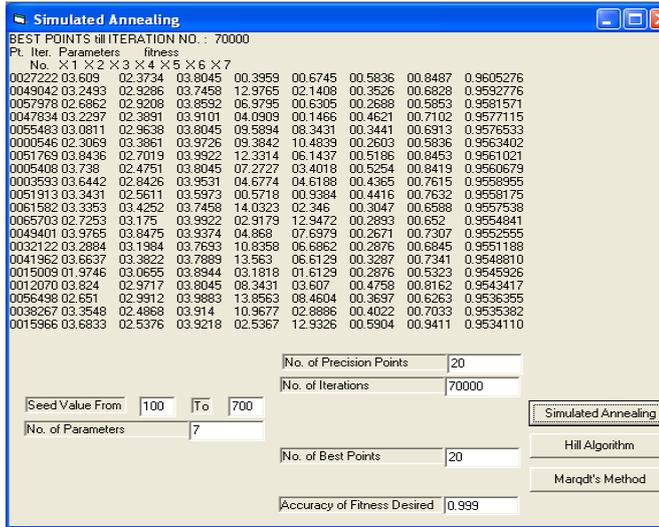
- (i) Number of best solutions=20
- (ii) Number of iteration=70000
- (iii) Number of precision points=20

Table II to Table VI depicts the details of Best 20 solution, Objective Function (Rad²), Structural error in degrees for the best 20 points. Fig.1to Fig.2 shows the best solution obtained by SA and SA/RS.

A. Results obtained by global Iteration (SA) for 20 accuracy points:

TABLE II

20 BEST POINTS FOR 20 ACCURACY POINTS –SA



The best desired accuracy of fitness for 20 accuracy points obtained by SA is **0.9605276**(first element of last column).

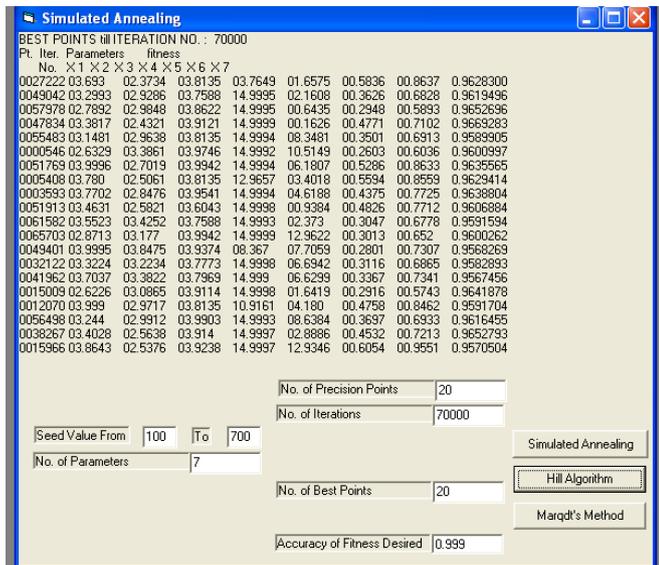
The best design vector is

$$\mathbf{X} = [3.609, 02.3734, 03.8045, 0.3959, 0.6754, 0.5836, 0.8487] \text{ (first row).}$$

B. Results obtained by hybrid Iteration for 20 accuracy points (SA/RS):

TABLE III

20 BEST POINTS FOR 20 ACCURACY POINTS SA/RS



The best desired accuracy of fitness for 20 accuracy points obtained by SA/RS is **0.9669283**(fourth element of last column).

The best design vector is

$$\mathbf{X} = [3.3817, 02.4321, 03.9121, 14.9999, 0.1626, 0.4771, 0.7102] \text{ (fourth row).}$$

TABLE IV

OBJECTIVE FUNCTION (RAD²) FOR 20 ACCURACY POINTS

Best Pts.	SA	SA/RS
1	0.041094498	0.038604946
2	0.042451111	0.039555503
3	0.043670187	0.03598
4	0.044155782	0.034202846
5	0.044219239	0.042763197
6	0.045653001	0.041558496
7	0.045913402	0.037821861
8	0.045950816	0.038484793
9	0.046139458	0.037473114
10	0.046224828	0.04092024
11	0.046294558	0.042579575
12	0.046589891	0.041638239
13	0.046840348	0.045121119
14	0.046990175	0.043526209
15	0.047250914	0.045209928
16	0.047567308	0.037142349
17	0.047842717	0.042567619
18	0.048618681	0.03988424
19	0.048725683	0.035969589
20	0.048865599	0.044877051

C. Mechanism Synthesis Solution for 20 Accuracy Points

TABLE V

STRUCTURAL ERROR WITH 20 ACCURACY POINTS (SA)

θ	ϕ_{EXP}	ϕ_{GEN}	$(\phi_{EXP}) - (\phi_{GEN})$
9	66.9462	72.1724	-5.2262(max.)
18	69.0114	73.5856	-4.5742
27	71.1093	73.8261	-2.7168
36	73.2425	72.8729	0.3696(min.)
45	75.4136	73.6560	1.7576
54	77.6252	75.0783	2.5469
63	79.8800	77.0332	2.8468
72	82.1804	79.4156	2.7648
81	84.5292	82.1274	2.4018
90	87.9290	85.0795	2.8495
99	89.3823	88.1911	1.1912
108	92.8918	91.3886	1.5032
117	94.4601	95.3537	-0.8936
126	97.0897	98.7727	-1.683
135	99.7833	100.8351	-1.0518
144	102.5435	103.7337	-1.1902
153	105.3729	106.4150	-1.0421
162	108.2742	109.8307	-1.5565
171	112.2498	110.939	1.3108
180	114.3025	112.7077	1.5948

TABLE VI

STRUCTURAL ERROR WITH 20 ACCURACY POINTS (SA/RS)

θ	ϕ_{EXP}	ϕ_{GEN}	$(\phi_{EXP}) - (\phi_{GEN})$
9	63.9019	68.4253	-4.5234(max.)
18	65.5521	68.0488	-2.4968
27	67.2211	68.2903	-1.0692
36	68.9105	68.1540	0.7565
45	69.6217	68.6048	1.0169
54	71.3561	69.5826	1.7735
63	72.1150	71.0139	1.1011
72	73.9000	72.8218	1.0782
81	75.7125	74.9308	0.7817
90	77.5539	77.2696	0.2843
99	80.4255	79.7714	0.6541
108	83.3290	82.3739	0.9551
117	85.2655	85.0175	0.248(min.)
126	87.2367	87.6453	-0.4086
135	89.2439	90.2016	-0.9577
144	91.2885	92.6317	-1.3432
153	93.3720	93.8842	-0.5122
162	95.4957	96.9108	-1.4151
171	97.6615	98.6699	-1.0084
180	99.8698	100.1291	-0.2593

VII. CONCLUDING REMARKS

The paper may be concluded with the following observations:

1. A hybrid optimization method based on the fusion of the SA and RS is developed and applied on MSP. A newly developed SA/RS is effective and fast in the solution of MSP. It also shows closer convergence properties.
2. The results are encouraging and suggest that SA/RS can be used effectively and efficiently in other complex and realistic design often encountered in engineering applications.
3. It can conclude from computer simulation results (Fig.2 and Fig3) that Minimum and Maximum error is greatly reduced by Hybrid iteration. Therefore SA/RS is an effective tool for MSP. The entire work may be concluded with the following observations.
4. Simulation results demonstrate the remarkable advantages of our approach in achieving the diverse optimal solutions and improved converge speed.
5. Hybrid algorithms have shown outstanding reliability and efficiency in application to the mechanism optimization problem.

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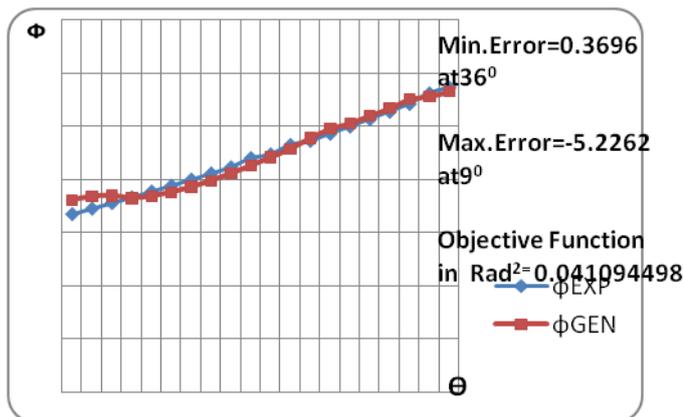


Fig.2 Best solution obtained by SA for 20 Accuracy Points

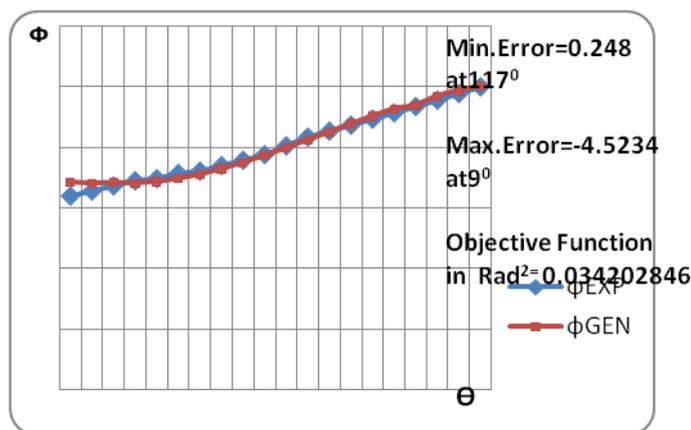


Fig. 3 Best solution obtained by SA/RS for 20 Accuracy Points

Simulation results Figs. (2-3) demonstrate the remarkable advantages of our approach in achieving the diverse optimal solutions and improved converge speed. It is clear that the hybrid approaches are effective and produce the good results for 20 accuracy points.

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